Equivalence Genesis and Recursive Ontogenesis: Obstruction-Driven Identity Policies Beyond Homotopy Equivalence and Univalence

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Mathematics has progressively enlarged its concept of sameness: congruence preserves metric form, topology preserves continuity, homotopy preserves deformation type, and univalence internalizes structural equivalence as identity. Yet each framework presupposes the equivalences permitted to count as identity. This paper introduces equivalence genesis, a formal program in which identity policies are constructed rather than merely selected. A failed identification produces an obstruction certificate; an admissible invariant is extracted from that obstruction; the invariant generates a transformation-stable equivalence relation or localization; and the resulting policy forms a new quotient or higher object universe. Semantic novelty is required: two policies are identical when their induced localization operators are naturally equivalent. Refinement, coarsening, lattice operations, contextual optimization, and higher localization together define an expandable algebra of identity policies. For finite state spaces, policy discovery terminates after finitely many extensional equivalence relations; open-ended novelty therefore requires expanding observations, transformations, representational spaces, or higher structure. Because newly formed objects support previously unavailable observations, equivalence genesis becomes recursively self-amplifying. We call this feedback process recursive ontogenesis: objects generate observations, observations expose obstructions, obstructions generate identities, and identities generate new objects.

Keywords: equivalence genesis, recursive ontogenesis, identity policies, obstruction certificates, admissible invariants, object formation, homotopy equivalence, Homotopy Type Theory, univalence, modalities, localization, quotient objects, observational equivalence, policy algebra, policy abiogenesis, ontological dynamics, variational objecthood, higher category theory, abstract algebra, mathematical discovery.

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0. Equivalence Genesis: The Endogenous Creation of Identity Policies

The previous theory assumed a fixed policy space:

$$\Pi = \{\pi_1, \pi_2, \dots\}.$$

A context selected the best available policy:

$$\pi^*(c) \in \arg\min_{\pi \in \Pi} \mathcal{J}(c, \pi).$$

But this cannot produce a genuinely new notion of identity. It can only choose among notions already supplied.

The next step is therefore:

$$\boxed{\text{ontology selection} \longrightarrow \text{equivalence genesis.}}$$

Equivalence genesis is the process by which failures of an existing ontology generate a new identity policy.

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1. The limitation of a fixed policy universe

Suppose every existing policy fails in some context c. This may occur because:

$$D(c, \pi) > \varepsilon$$

for every $\pi \in \Pi$, where D measures task-relevant distortion.

Alternatively, every policy may violate stability:

$$S(c, \pi) \neq 0,$$

or produce a nontrivial obstruction:

$$\Omega(c, \pi) \not\simeq *.$$

Then the admissible set is empty:

$$\Pi_{\mathrm{adm}}(c) = \emptyset.$$

Optimization over Π is no longer enough. A new policy must be constructed:

$$\eta \notin \Pi.$$

The theory must therefore contain a policy generator.

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2. Policies as kernels of observation

Let X be a raw state space and let

$$h: X \longrightarrow Y$$

be an observation.

The observation induces an equivalence relation:

$$x \equiv_h y \Leftrightarrow h(x) = h(y).$$

This is the kernel relation of h.

For a family of observations

$$\mathcal{H} = \{h_\alpha: X \to Y_\alpha\},$$

define

$$x \equiv_{\mathcal{H}} y \Leftrightarrow h_\alpha(x) = h_\alpha(y)$$

for every α.

Thus, every collection of admissible observations generates an identity policy:

$$\boxed{\mathcal{H} \longmapsto \pi_{\mathcal{H}}.}$$

The creation of a new policy can therefore be reduced to the creation of a new invariant, observation, or admissible test.

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3. Transformation-stable policy generation

Let $\mathcal{G}$ be a monoid of admissible transformations of X.

A transformation-stable policy generated by $\mathcal{H}$is defined by

$$x \equiv_{\mathcal{H},\mathcal{G}} y \Leftrightarrow h(gx) = h(gy)$$

for every $h \in \mathcal{H}$ and $g \in \mathcal{G}$.

This ensures

$$x \equiv_{\mathcal{H},\mathcal{G}} y \Longrightarrow kx \equiv_{\mathcal{H},\mathcal{G}} ky$$

for every $k \in \mathcal{G}$.

The new policy is therefore not merely a partition of X. It is a partition compatible with the lawful transformations recognized by the context.

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4. The obstruction-to-invariant principle

Suppose an existing policy πidentifies two states:

$$x \equiv_{\pi} y,$$

but the context reveals that the identification is defective.

Represent the failure by an obstruction:

$$\omega \in \Omega_{\pi}(x, y).$$

A policy generator attempts to extract an invariant

$$I_{\omega}: X \longrightarrow Z$$

such that

$$I_{\omega}(x) \neq I_{\omega}(y).$$

The policy can then be refined:

$$\pi' = \pi \wedge \ker(I_\omega).$$

Explicitly,

$$u \equiv_{\pi'} v \iff u \equiv_\pi v \text{ and } I_\omega(u) = I_\omega(v).$$

The obstruction has been converted into a new admissible distinction.

This gives the central generative cycle:

failed identity → obstruction → new invariant → new policy → new objects.

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5. Obstruction-Driven Refinement Proposition

Let πbe a transformation-stable equivalence relation on X. Let

$$I: X \longrightarrow Z$$

be an invariant satisfying

$$I(gu) = I(gv)$$

whenever $I(u) = I(v)$, for every admissible transformation g.

Define

$$u \equiv_{\pi'} v \iff u \equiv_\pi v \text{ and } I(u) = I(v).$$

Then:

1. π' is an equivalence relation.
2. π' is transformation-stable.
3. π' refines π.
4. If some pair satisfies

$$u \equiv_\pi v \text{ but } I(u) \neq I(v),$$

then the refinement is strict.

Thus, every admissible obstruction-separating invariant generates a mathematically valid new identity policy.

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6. Coarsening from irrelevant distinctions

Policy generation need not always refine an ontology.

A context may reveal that an existing invariant is irrelevant. Suppose a policy is generated by

$$\mathcal{H} = \{h_1, \dots, h_n\}.$$

If h_n is no longer admissible, define

$$\mathcal{H}' = \{h_1, \dots, h_{n-1}\}.$$

Then

$$\pi_{\mathcal{H}} \preceq \pi_{\mathcal{H}'}.$$

Removing an invariant creates a coarser policy.

Thus, there are two basic forms of policy genesis:

refinement	:add an admissible distinction,
coarsening	:remove an inadmissible distinction.

A third form combines existing distinctions in a new way rather than merely adding or deleting them.

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7. The algebra of policy construction

Let $\pi$and ρbe two identity policies.

Their common refinement is

$$\pi \wedge \rho,$$

defined by

$$x \equiv_{\pi \wedge \rho} y \Leftrightarrow x \equiv_{\pi} y \text{ and } x \equiv_{\rho} y.$$

Their common coarsening is

$$\pi \vee \rho,$$

the smallest equivalence relation containing both $\pi$and ρ.

Policy generation may use operations such as:

$$\pi, \rho \longmapsto \pi \wedge \rho,$$
$$\pi, \rho \longmapsto \pi \vee \rho,$$
$$\pi \longmapsto \mathrm{Stab}_{\mathcal{G}}(\pi),$$
$$\{\pi_i\} \longmapsto \mathrm{Glue}(\{\pi_i\}),$$
$$\pi \longmapsto \mathrm{Complete}(\pi).$$

These operations create a policy algebra.

However, closure under these operations only generates combinations of existing distinctions. Genuine open-ended policy genesis requires the introduction of newly constructed invariants.

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8. Policy novelty

A syntactically new policy need not be mathematically new.

Two different descriptions may generate the same resolved objects:

$$L_{\pi} \simeq_{\mathrm{nat}} L_{\rho}.$$

Define semantic policy equivalence by

$$\pi \simeq_{\Pi} \rho \Leftrightarrow L_{\pi} \simeq_{\mathrm{nat}} L_{\rho}.$$

A candidate policy $\eta$is genuinely new relative to Π when

$$\boxed{\forall \pi \in \Pi, L_{\eta} \not\simeq_{\mathrm{nat}} L_{\pi}.}$$

This prevents mere renaming, reformulation, or redundant implementation from counting as policy birth.

The new policy must induce a genuinely different notion of objecthood.

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9. The admissibility criterion for policy birth

Let $\mathrm{Gen}(\Pi, \Omega, c)$ be the family of policies generated from:

- the current policy repertoire Π;
- the observed obstruction structure Ω;
- the current context c.

Define the admissible candidate set:

$$\mathcal{A}(c) = \left\{ \eta \in \mathrm{Gen}(\Pi, \Omega, c) : \begin{matrix} D(c, \eta) \leq \varepsilon, \\ S(c, \eta) = 0, \\ G(c, \eta) = 0, \\ \Omega(c, \eta) \simeq * \end{matrix} \right\}.$$

Here:

- D is task-relevant distortion;
- S is transformation instability;
- G is gluing incoherence;
- Ω is residual obstruction.

A new policy is selected by minimum complexity:

$$\eta^*(c) \in \underset{\eta \in \mathcal{A}(c)}{\arg\min}\, K(\eta).$$

Policy space then expands:

$$\boxed{\Pi_{t+1} = \mathrm{Cl}(\Pi_t \cup \{\eta^*(c_t)\}).}$$

The closure operator adds all policy constructions that are automatically implied by the new policy.

10. The equivalence-genesis equation

The entire process can be written as

$$\boxed{\begin{array}{cc} \omega_t & \in \Omega_{\pi_t}(X_t, c_t), \\ I_t & = \mathrm{Extract}(\omega_t), \\ \eta_t & = \mathrm{Policy}(I_t, \mathcal{G}_t), \\ \Pi_{t+1} & = \mathrm{Cl}(\Pi_t \cup \{\eta_t\}), \\ \mathfrak{O}_{t+1}(c) & = L_{\pi^*_{t+1}(c)} X_c. \end{array}}$$

This is the proposed mechanism of equivalence genesis.

An ontology fails. Its failure yields an obstruction. The obstruction generates a new invariant. The invariant generates a new identity policy. The new identity policy generates new objects.

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11. Finite Policy-Genesis Theorem

Let X be a finite set with

$$| X | = n.$$

Assume every identity policy is interpreted extensionally as an equivalence relation on X. Assume every successful policy birth adds a relation not equivalent to any previously generated relation.

Then the process must terminate after finitely many policy births.

The number of equivalence relations on an n-element set is the Bell number

$$B_n.$$

Therefore, starting from a repertoire Π_0, the number of semantically new policy births is at most

$$B_n - | \Pi_0 |.$$

This establishes:

$$\boxed{\text{Finite state spaces permit discovery, but not unbounded policy novelty.}}$$

Open-ended equivalence genesis requires at least one of the following:

- an unbounded state space;
- an expanding observation language;
- new transformations;
- new contexts;
- higher structure beyond set-level partitions;
- recursive construction of new representational spaces.

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12. Triangle example: discovering similarity

Let X be the space of nondegenerate Euclidean triangles.

Suppose the initial policy is congruence:

$$T \equiv_{\text{cong}} T'$$

when corresponding side lengths are equal.

Now introduce a context in which global scale is irrelevant. The existing policy treats

$$(3, 4, 5)$$

and

$$(6, 8, 10)$$

as different objects, although every task-relevant relation is preserved.

The obstruction is unwanted sensitivity to scale.

Extract the normalized side invariant:

$$I(T) = \frac{(a, b, c)}{a + b + c}.$$

Then define

$$T \equiv_{\text{sim}} T' \Leftrightarrow I(T) = I(T').$$

The newly generated policy is similarity.

The ontology changes from individual metric triangles to shape classes:

$$\boxed{\text{congruence classes} \longrightarrow \text{similarity classes.}}$$

Historically, similarity is familiar. Formally, however, the example demonstrates how a new equivalence can be generated from an obstruction to an existing identity policy.

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13. Platonic solids: generating an intermediate ontology

Under a rigid geometric policy, the five Platonic solids remain distinct:

$$5.$$

Under boundary topology, all five become copies of the sphere:

$$1.$$

The transition

$$5 \longrightarrow 1$$

may be too destructive for a context that regards duality as meaningful.

A new invariant can be extracted from the interchange of faces and vertices.

Polyhedral duality gives:

$$\begin{aligned} &\text{tetrahedron} \longleftrightarrow \text{tetrahedron},\\ &\text{cube} \longleftrightarrow \text{octahedron},\\ &\text{dodecahedron} \longleftrightarrow \text{icosahedron}. \end{aligned}$$

Define

$$P \equiv_{\text{dual}} Q$$

when P is isomorphic to Qor to the dual of Q.

The resulting ontology has three classes:

{tetrahedron},
{cube,octahedron},
{dodecahedron,icosahedron}.

Thus:

$$\boxed{5 \longrightarrow 3 \longrightarrow 1.}$$

The intermediate policy is not produced by simple metric forgetting alone. It is generated by recognizing a structural transformation—duality—as identity-producing.

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14. Generating higher policies by localization

In Homotopy Type Theory, a policy may be represented by a localization or modality

$$L: \mathcal{U} \longrightarrow \mathcal{U}.$$

A localization can be generated by selecting a family of maps

$$S = \{f_\alpha : A_\alpha \to B_\alpha\}$$

that are to become equivalences.

The resulting localization is written schematically as

$$L_S = \mathrm{Loc}(S).$$

An obstruction may reveal that a map

$$f : A \to B$$

should count as an equivalence but does not under the current modality.

The policy generator adds it to the localization data:

$$S_{t+1} = S_t \cup \{f\}.$$

Then

$$L_{t+1} = \mathrm{Loc}(S_{t+1}).$$

The higher equivalence-genesis equation is therefore

$$\boxed{L_{t+1} = \mathrm{Loc}(S_t \cup \mathrm{Maps}(\Omega_t)).}$$

The obstruction determines which maps must newly become invertible.

This gives a constructive route from ontological failure to a new homotopical identity policy.

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15. The policy-discovery machine

A minimal computational system would perform the following cycle.

1. Apply the current policy to the raw state space.
2. Test the resulting objects under admissible transformations and tasks.
3. Locate failures of prediction, stability, compression, or gluing.
4. encode each failure as an obstruction certificate;
5. Search for an invariant that detects or resolves the obstruction.
6. Construct a candidate equivalence relation from the invariant.
7. Stabilize the relation under admissible transformations.
8. Compare the candidate with all known policies semantically.
9. Reject redundant or inadmissible candidates.
10. Add a genuinely new minimal policy to the repertoire.
11. Recompute the object system.

This differs from ordinary classification because the classes are not fixed.

It differs from ordinary model selection because the hypothesis space itself can expand.

It differs from standard theorem discovery because the discovered entity is not merely a proposition.

The discovered entity is a new meaning of equality.

16. Policy abiogenesis

The strongest form of the theory would begin without named equivalences such as:

- congruence;
- similarity;
- homeomorphism;
- homotopy equivalence;
- isomorphism;
- duality.

It would begin only with:

- raw states;
- primitive transformations;
- primitive observations;
- failures;
- admissibility rules.

Through repeated obstruction-driven construction, stable equivalence relations would emerge.

This is:

policy abiogenesis.

Algebraic abiogenesis seeks the emergence of laws from anonymous operations.

Policy abiogenesis seeks the emergence of identity from anonymous distinctions.

The two programs can be joined:

operations generate laws and obstructions generate identities.

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17. Self-application

The policy generator itself uses a notion of when two policies are equivalent.

Therefore, equivalence genesis eventually becomes self-referential.

At the first level:

$$\pi$$

determines identity among objects.

At the second level:

$$\mu$$

determines identity among policies.

At the third level:

$$\nu$$

determines identity among meta-policies.

This produces a hierarchy:

$$\Pi_0 \longrightarrow \Pi_1 \longrightarrow \Pi_2 \longrightarrow \cdots$$

where

$$\Pi_{k+1}$$

contains identity policies for the structures in Π_k.

To avoid unrestricted self-reference, the hierarchy should be stratified by universe level.

The long-term objective is a coherent limit in which object formation and policy formation become mutually recursive.

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18. Recursive ontogenesis

Once newly generated policies produce new objects, those objects may support observations that were previously impossible.

New objects therefore create new invariants.

New invariants create new policies.

New policies create further objects.

The feedback loop is

objects → observations → obstructions → policies → new objects.

This is recursive ontogenesis.

The ontology is no longer merely selected or changed. It expands its own capacity to make distinctions.

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19. The central principle

The proposed theory can be condensed to:

A new identity policy is born when an unresolved obstruction is converted into a stable, minimal, nonredundant invariant.

Formally:

$$\eta^* = \arg\min_{\eta} K(\eta)$$

subject to

$$D(c,\eta) \leq \varepsilon,$$
$$S(c,\eta) = 0,$$
$$G(c,\eta) = 0,$$
$$\Omega(c,\eta) \simeq *,$$

and

$$L_{\not\eta} \simeq_{\mathrm{nat}} L_{\pi}$$

for every previously known policy π.

This is stronger than choosing an equivalence relation.

It specifies the conditions under which a new form of equivalence deserves to enter mathematics.

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20. The extended ladder

The sequence now becomes:

Triangles	→ metric identity,
Platonic solids	→ structural classification,
Topology	→ continuous identity,
Homotopy equivalence	→ deformational identity,
Univalence	→ equivalence as identity,
Variational objecthood	→ selection of identity,
Ontological dynamics	→ change of identity,
Ontology discriminants	→ geometry of identity transitions,
Equivalence genesis	→ creation of identity policies,
Recursive ontogenesis	→ self-expanding object formation.

The next arrow is therefore:

homotopy equivalence → univalence → identity genesis → equivalence genesis → recursive ontogenesis.

At this point, mathematics no longer merely studies objects, classifies them, or declares equivalent objects identical.

It begins to explain how previously unavailable forms of sameness can come into existence.

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